Stat310

Discrete random variables

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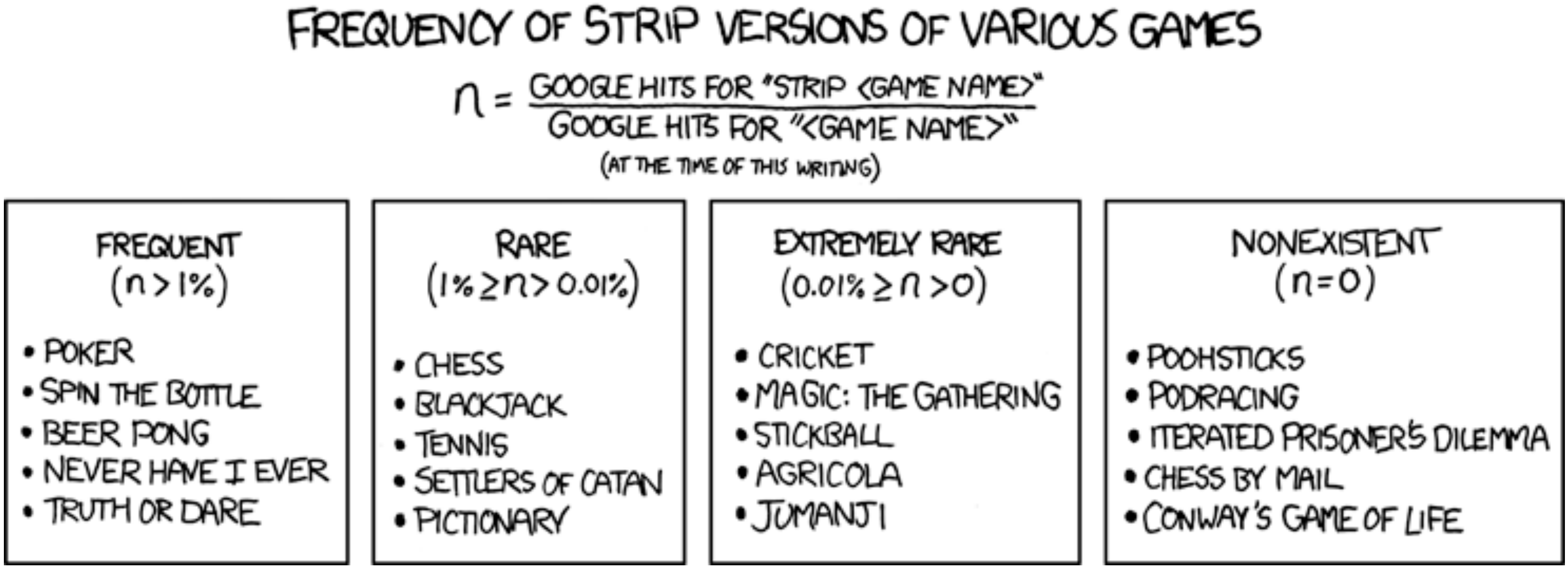
Quiz

- Pick up quiz on your way in
- Start at 1pm
- Finish at 1:10pm
- Closed book

Quiz
Due 1:10pm



http://xkcd.com/696/

1. Quiz!
2. Feedback
3. Discrete uniform distribution
4. More on the binomial distribution
5. Poisson distribution

What you like

- half–complete proofs
- video lecture
- examples/applications
- entertaining/engaging
- your turn
- homework
- help sessions
- website

What you'd like changed:

———————— more examples/applications

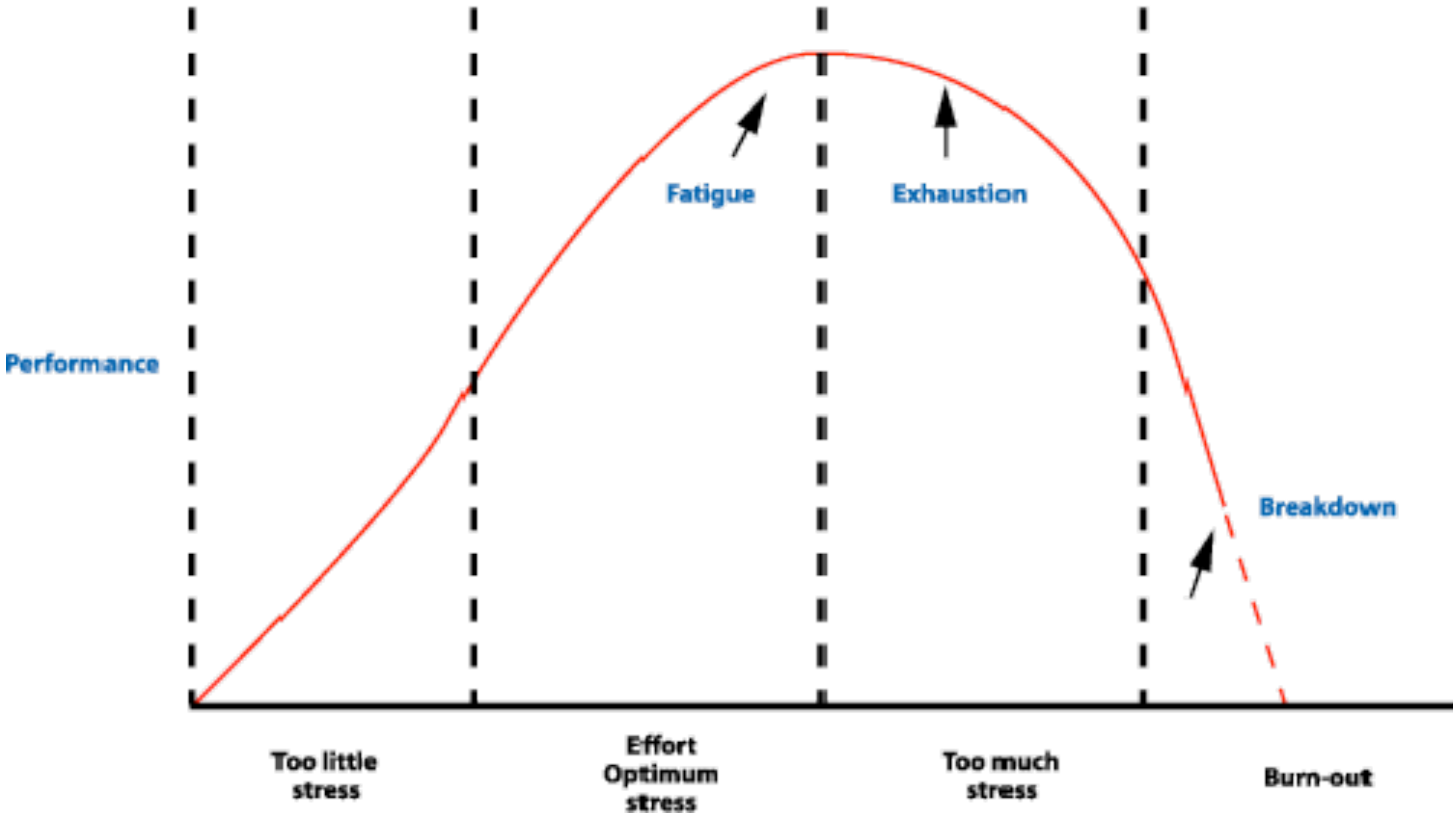
—————— go slower

————— **calling on people randomly**

——— less sex

—— owlspace

The Human Performance Curve



+		Δ
————————	homework	———
——————	help sessions	———
—————	reading	———————————————
——	practice problems	————
——	notes	
——	going to class	-

Slower, more details and basic math practice: http://khanacademy.org/

Discrete uniform

Discrete uniform

Equally likely events

$f(x) = 1/(b - a)$ $x = a, ..., b$

$X \sim \text{DiscreteUniform}(a, b)$

What is the mean?

What is the variance?

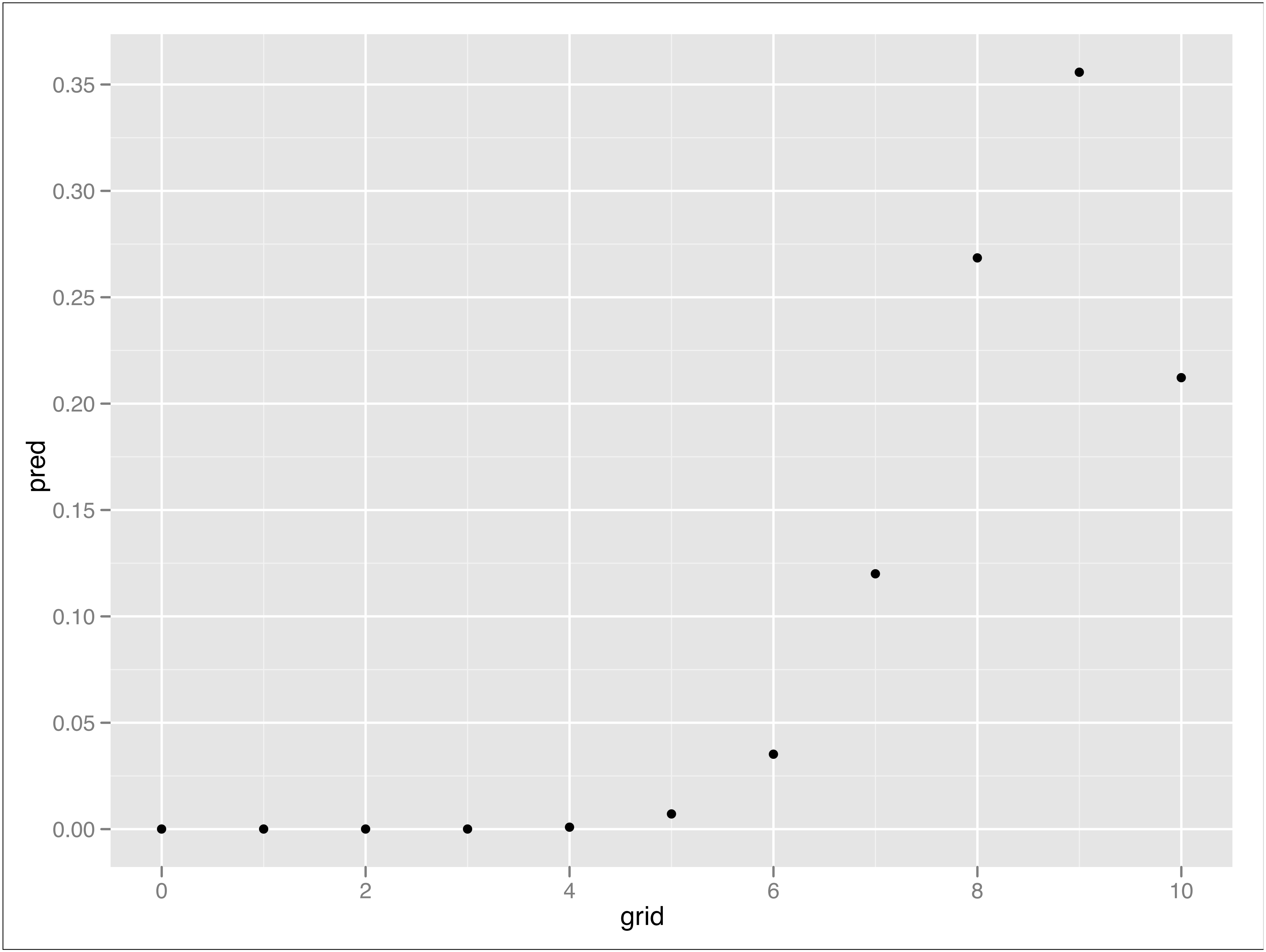
Binomial distribution

Your turn

In the 2008–09 season, Kobe Bryant attempted 564 field goals and made 483 of them.

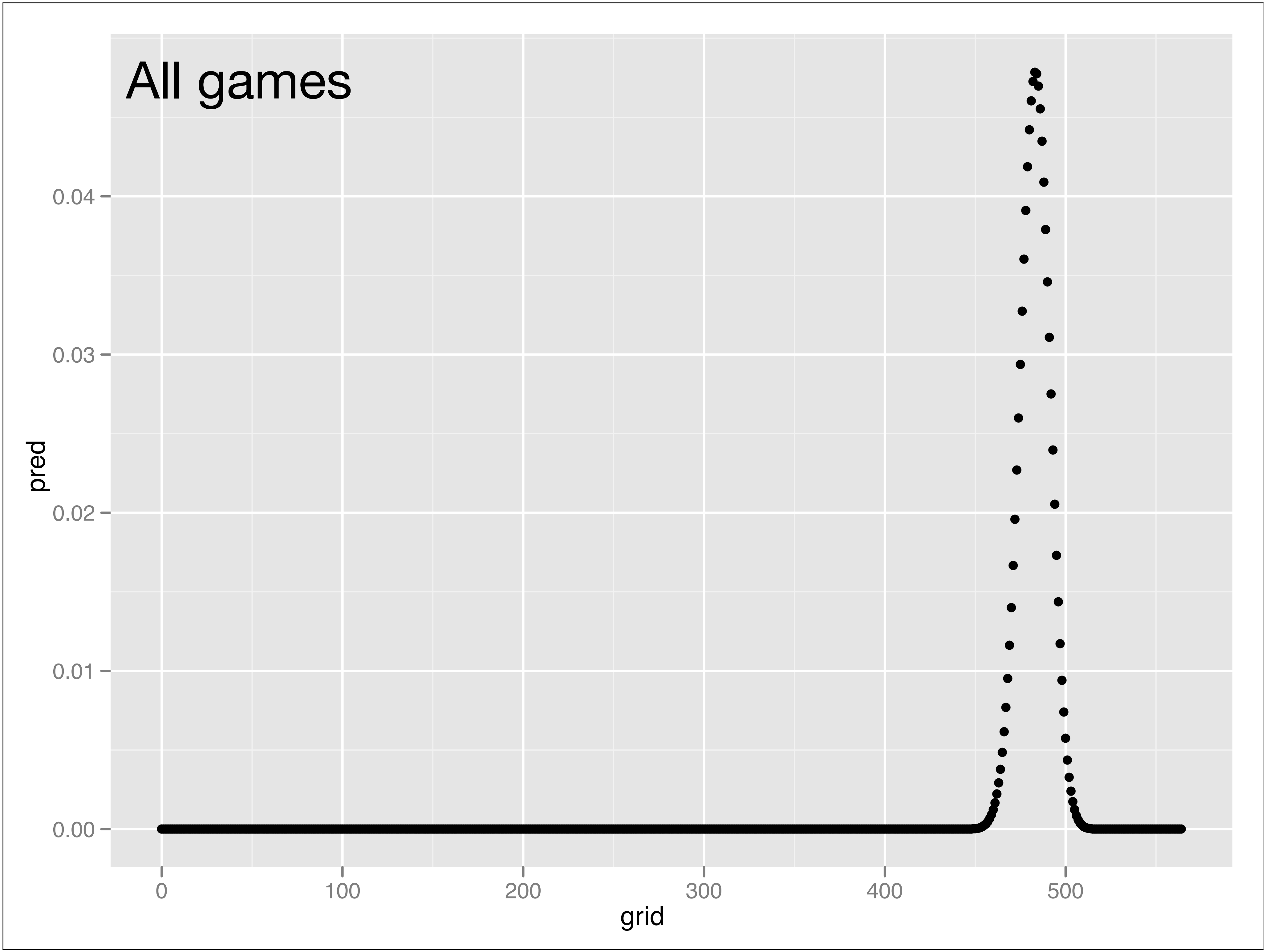
In the first game of the next season he makes 7 attempts. How many do you expect he makes? What's the probability he doesn't make any? What's the probability he makes them all?

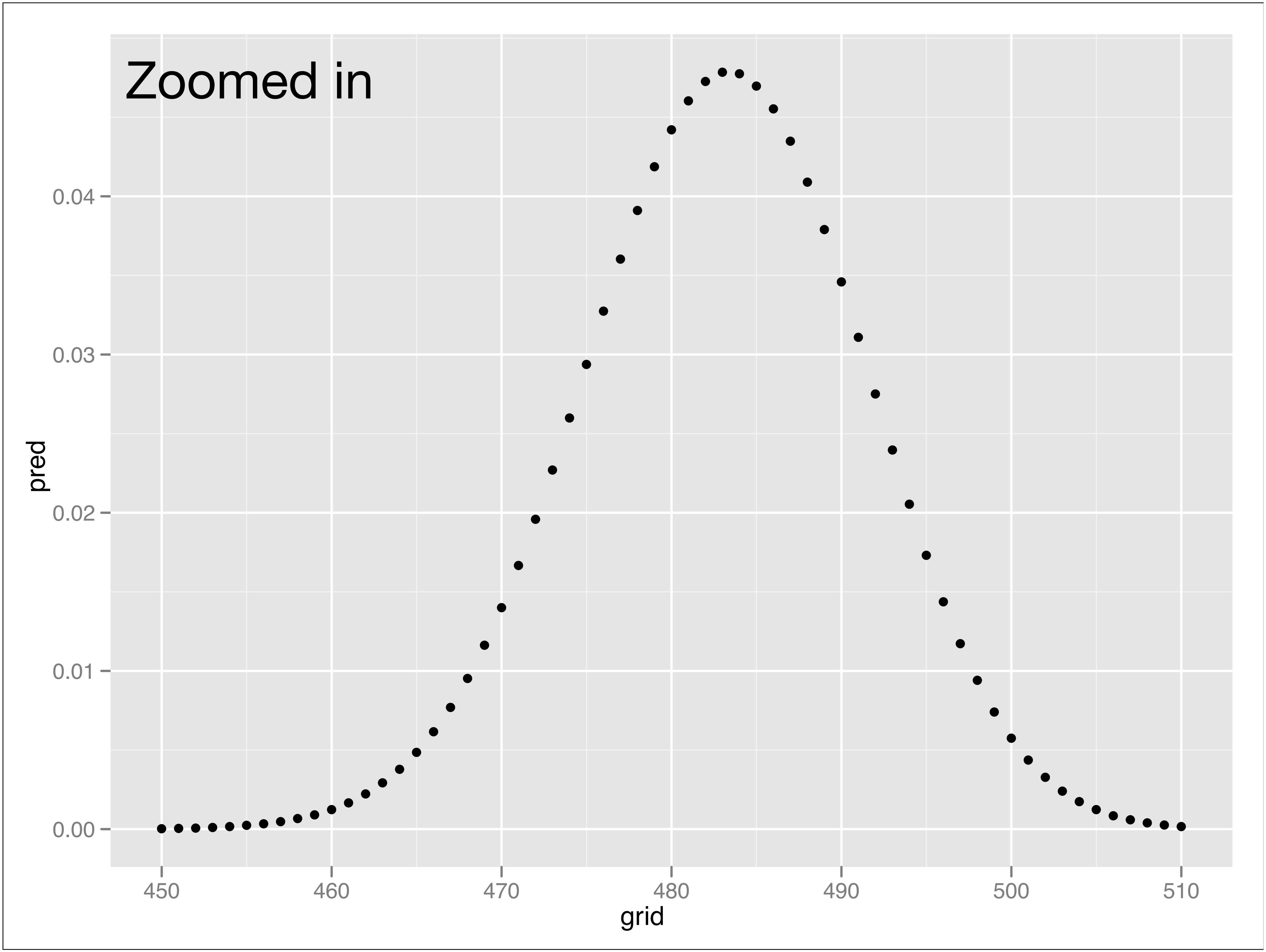
What assumptions did you make?



0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
pred
0
2
4
6
8
10
grid

What if we wanted to do the same thing for all games?


All games
pred
0.00
0.01
0.02
0.03
0.04
0
100
200
300
400
500
grid



Zoomed in
pred
0.00
0.01
0.02
0.03
0.04
grid
450
460
470
480
490
500
510

Sums

$X \sim \text{Binomial}(n, p)$

$Y \sim \text{Binomial}(m, p)$

$Z = X + Y$

Then $Z \sim$?

Poisson distribution

3.2.2 p. 119

Conditions

X = Number of times some event happens

(1) If number of events occurring in non–overlapping times is **independent**, and

(2) probability of exactly one event occurring in short interval of length h is $\propto$ λh, and

(3) probability of two or more events in a sufficiently short internal is basically 0

Poisson

$X \sim \text{Poisson}(\lambda)$

Sample space: positive integers

$\lambda \in [0, \infty)$

Arises as limit of binomial distribution when np is fixed and $n \to \infty$ = law of rare events.

Examples

Number of alpha particles emitted from a radioactive source

Number of calls to a switchboard

Number of eruptions of a volcano

Number of points scored in a game

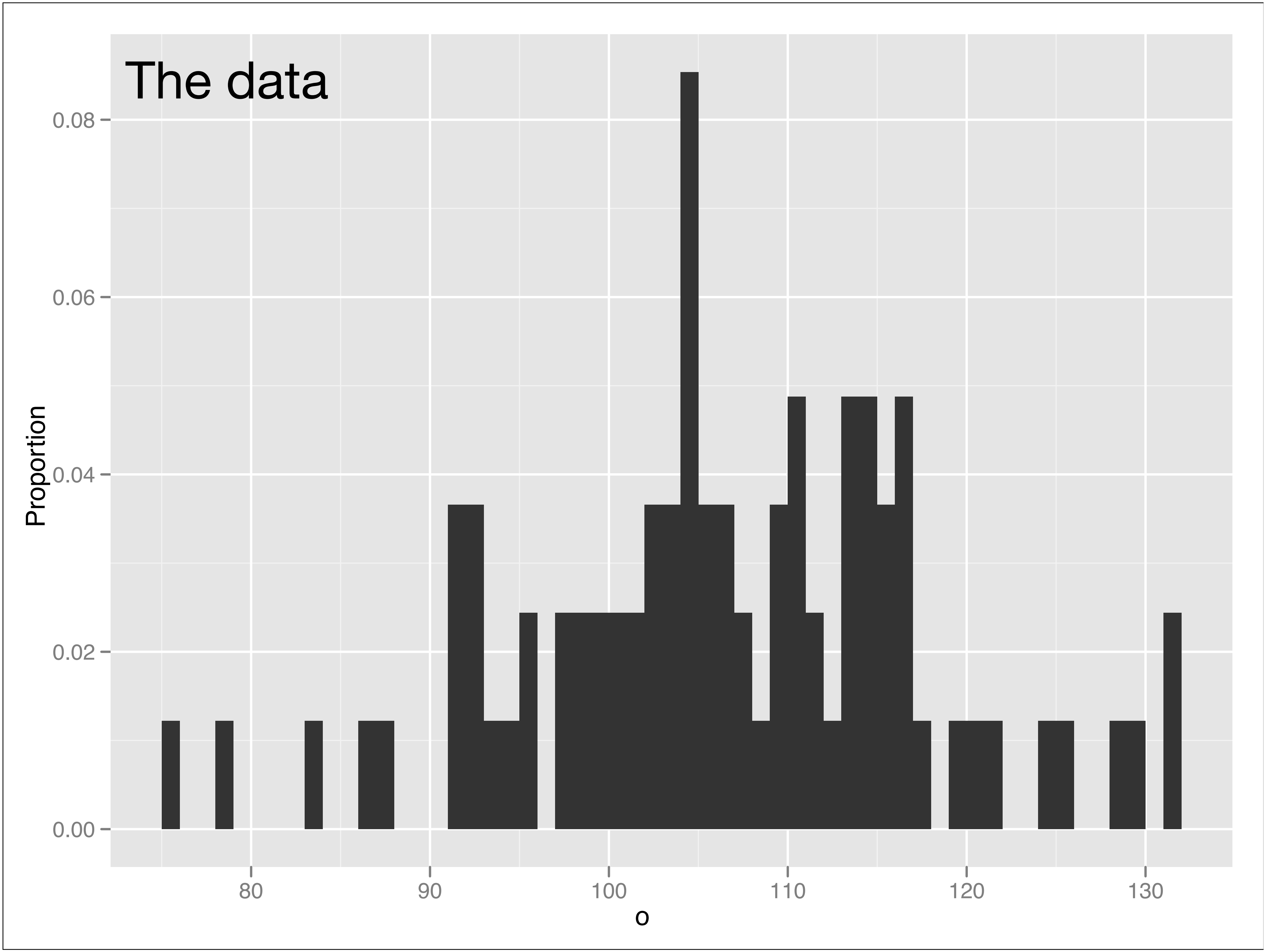
Basketball

Many people use the poisson distribution to model scores in sports games. For example, last season, on average the LA lakers scored 109.6 points per game, and had 101.8 points scored against them. So:

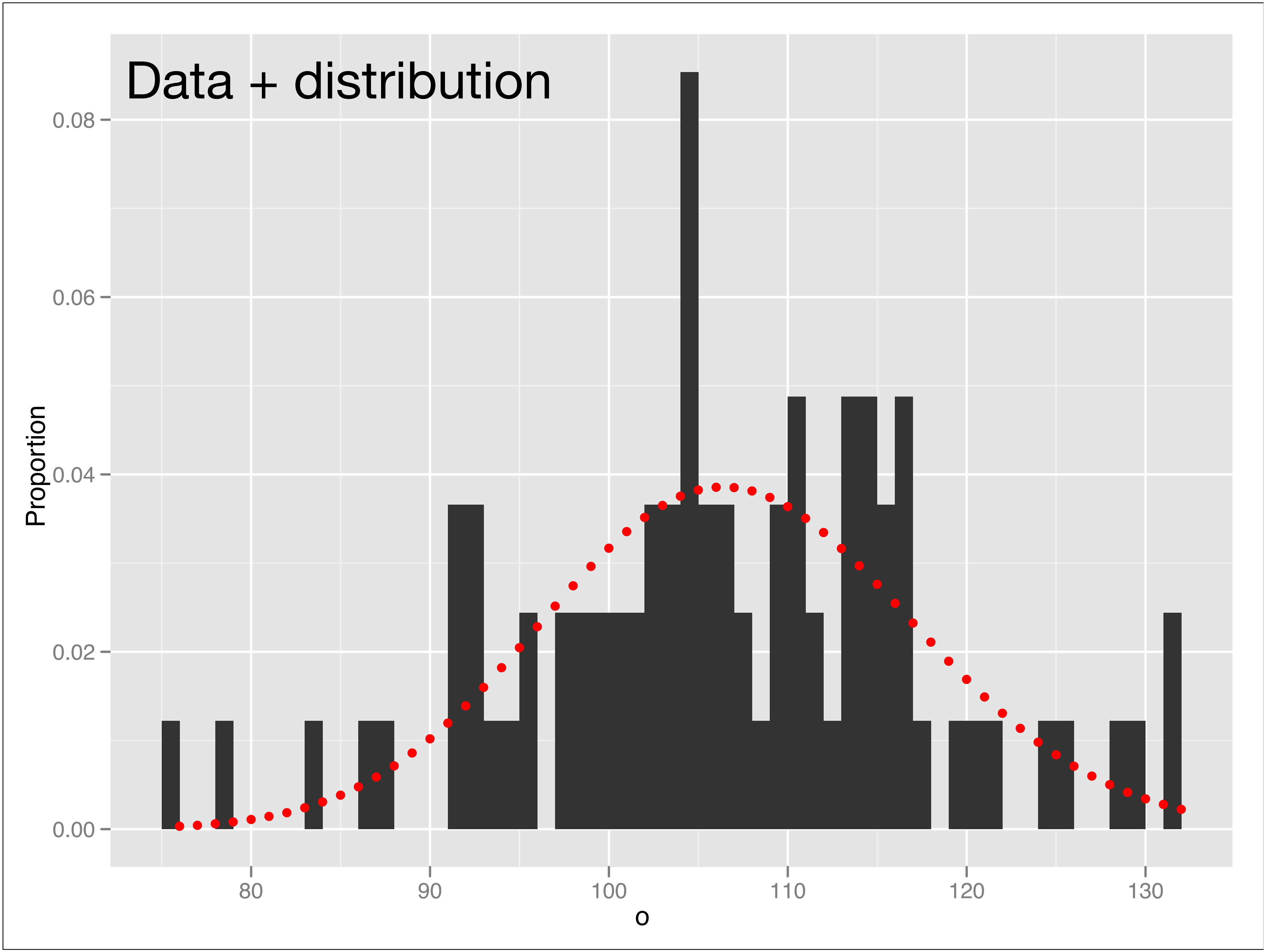
$O \sim \text{Poisson}(109.6)$
$D \sim \text{Poisson}(101.8)$

How can we check if this is reasonable?

http://www.databasebasketball.com/teams/teamyear.htm?tm=LAL&lg=n&yr=2008



The data
Proportion
0.00
0.02
0.04
0.06
0.08
80
90
100
110
120
130
o

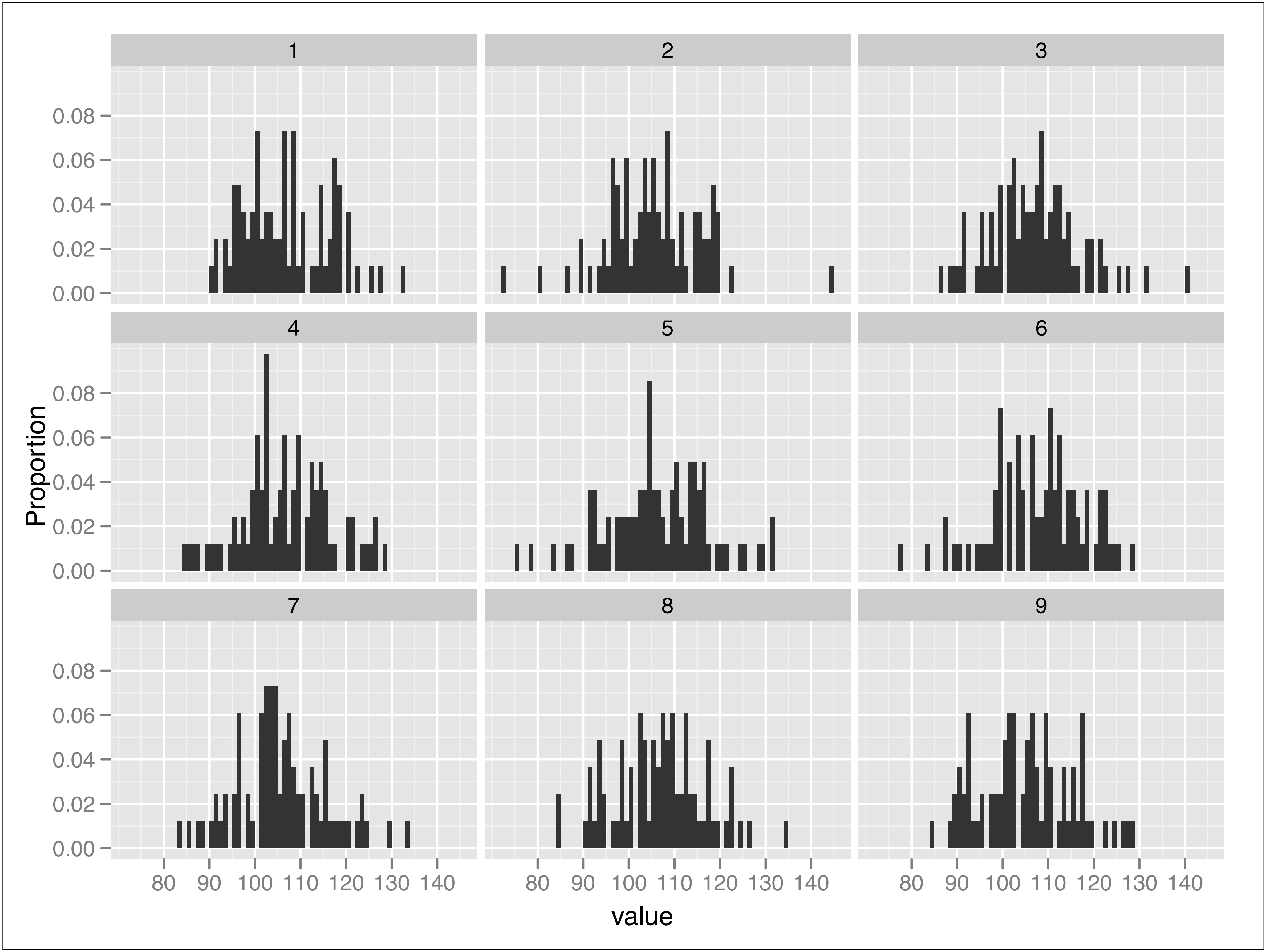


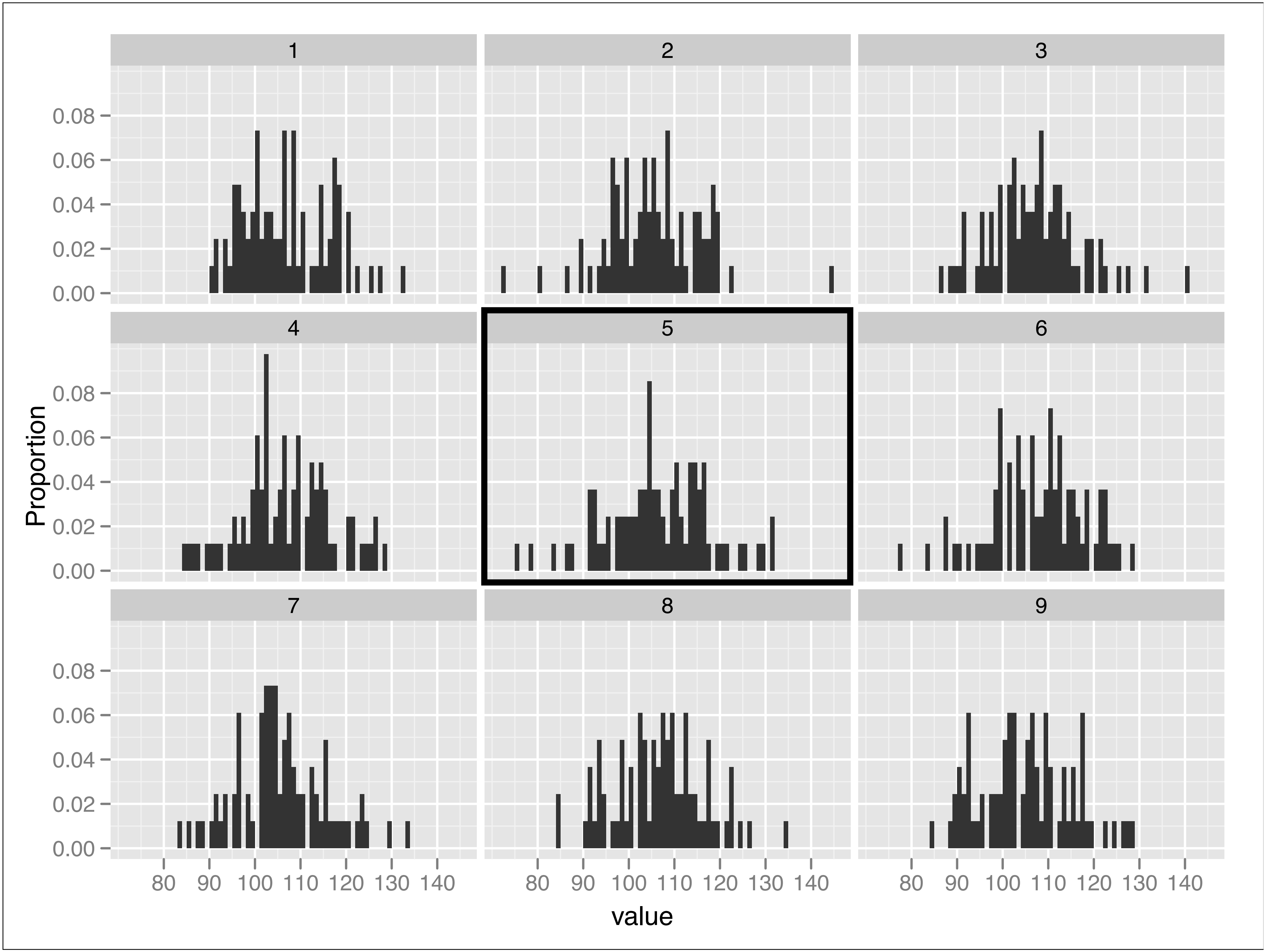
Data + distribution
Proportion
0.08
0.06
0.04
0.02
0.00
80
90
100
110
120
130
o

Simulation

Comparing to underlying distribution works well if we have a very large number of trials. But only have 108 here.

Instead we can randomly draw 108 numbers from the specified distribution and see if they look like the real data.


1
2
3
4
5
6
7
8
9
0.00
0.02
0.04
0.06
0.08
Proportion
80
90
100
110
120
130
140
value


1
2
3
4
5
6
7
8
9
Proportion
0.00
0.02
0.04
0.06
0.08
80
90
100
110
120
130
140
value

But

The distribution can’t be poisson. Why?

Example

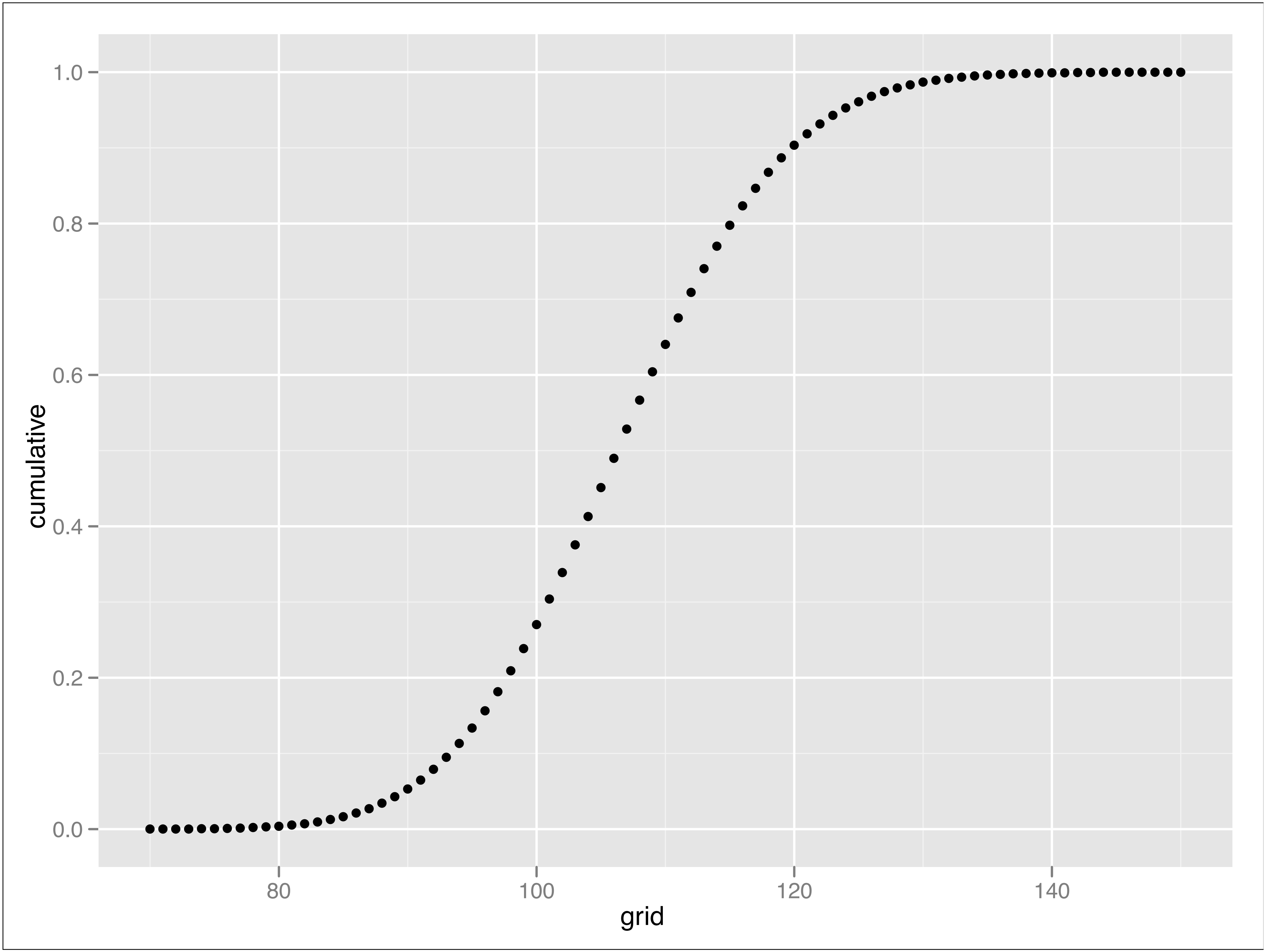
O ~ Poisson(109.6). D ~ Poisson(101.8)

On average, do you expect the Lakers to win or lose? By how much?

What's the probability that they score exactly 100 points?

Challenge

What's the probability they score over 100 points? (How could you work this out?)



cumulative
1.0
0.8
0.6
0.4
0.2
0.0
80
100
120
140
grid

Score difference

What about if we're interested in the winning/losing margin (offence – defence)?

What is the distribution of O – D?

It's not trivial to determine. We'll learn more about it when we get to transformations.

Multiplication

$X \sim \text{Poisson}(\lambda)$

$Y = tX$

Then $Y \sim \text{Poisson}(\lambda t)$

Summary

For a new distribution:

Compute expected value and variance from definition (given partial proof to complete)

Compute the mgf (given random mathematical fact)

Compute the mean and variance from the mgf (remembering variance isn't second moment)

Recognise

Discrete uniform

Bernoulli

Binomial

Poisson

Thursday

Continuous random variables.

New conditions & new definitions.

New distributions: uniform, exponential, gamma and normal

Read: 3.2.3, 3.2.5