Stat310

Moments

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Engineer Your Career

Monday, February 15
7:00 PM - 8:30 PM
McMurtry Auditorium

Find out what you can do with a degree in engineering from a panel of successful Rice engineering graduates who have gone into a variety of professions. (Plus get dessert!)

http://engineering.rice.edu/EventsList.aspx?EventRecord=13137

Homework

Due today.

From now on, if late, put in Xin Zhao's mail box in the DH mailroom.

Another one due next Thursday

Buy a stapler

Use official name

1. Finish off proof
2. More about expectation
3. Variance and other moments
4. The moment generating function
5. The Poisson distribution
6. Feedback

Proof, continued

Expectation
of a function

Expectation

Expectation is a **linear operator**:

Expectation of a sum =
sum of expectations (additive)

Expectation of a constant * a function =
constant * expectation of function (homogenous)

Expectation of a constant is a constant.

T 2.6.2 p. 95

Your turn

Write (or recall) the mathematical description of these properties.

Work in pairs for two minutes.

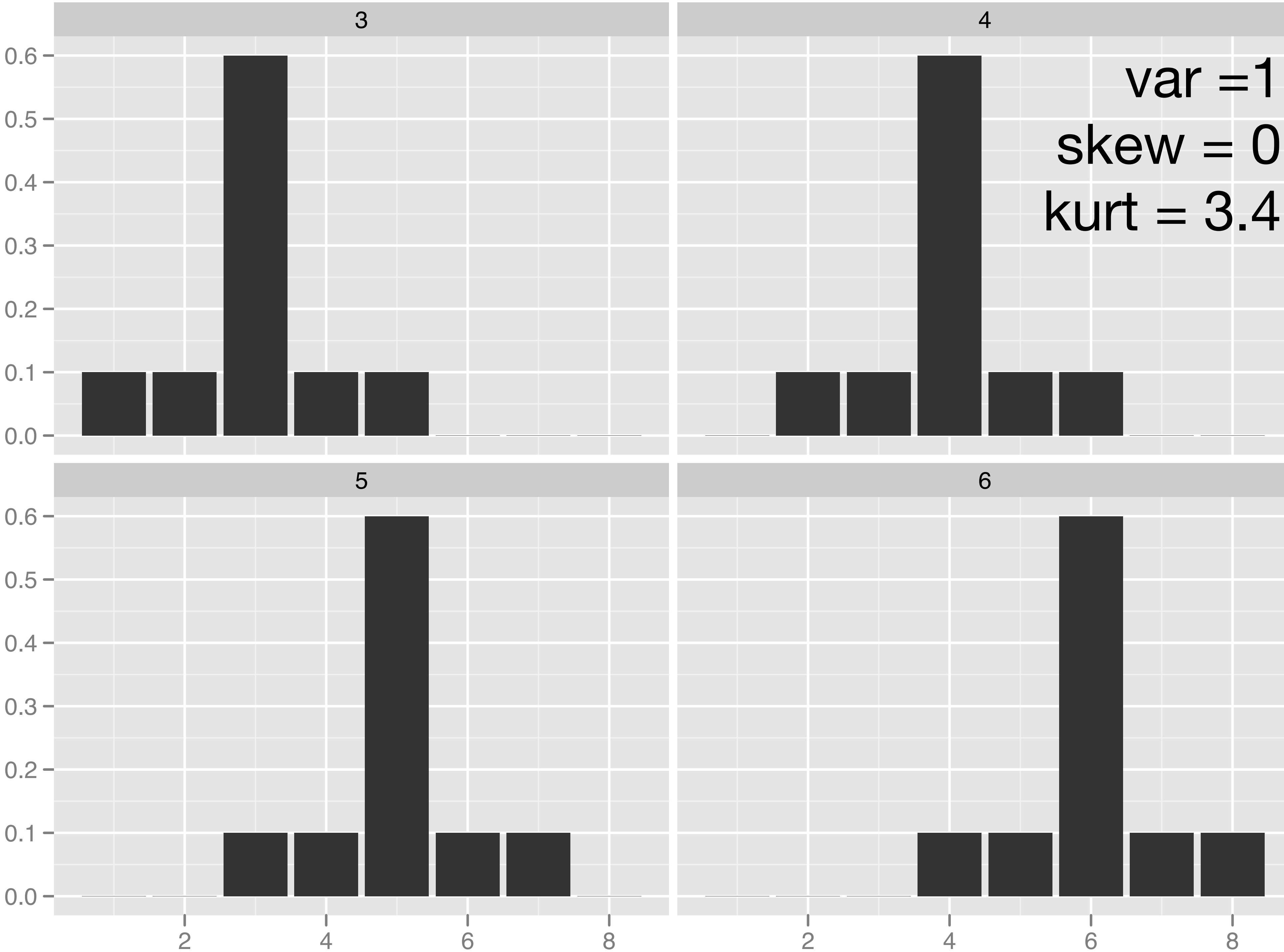
(Extra credit this week is to prove these properties)

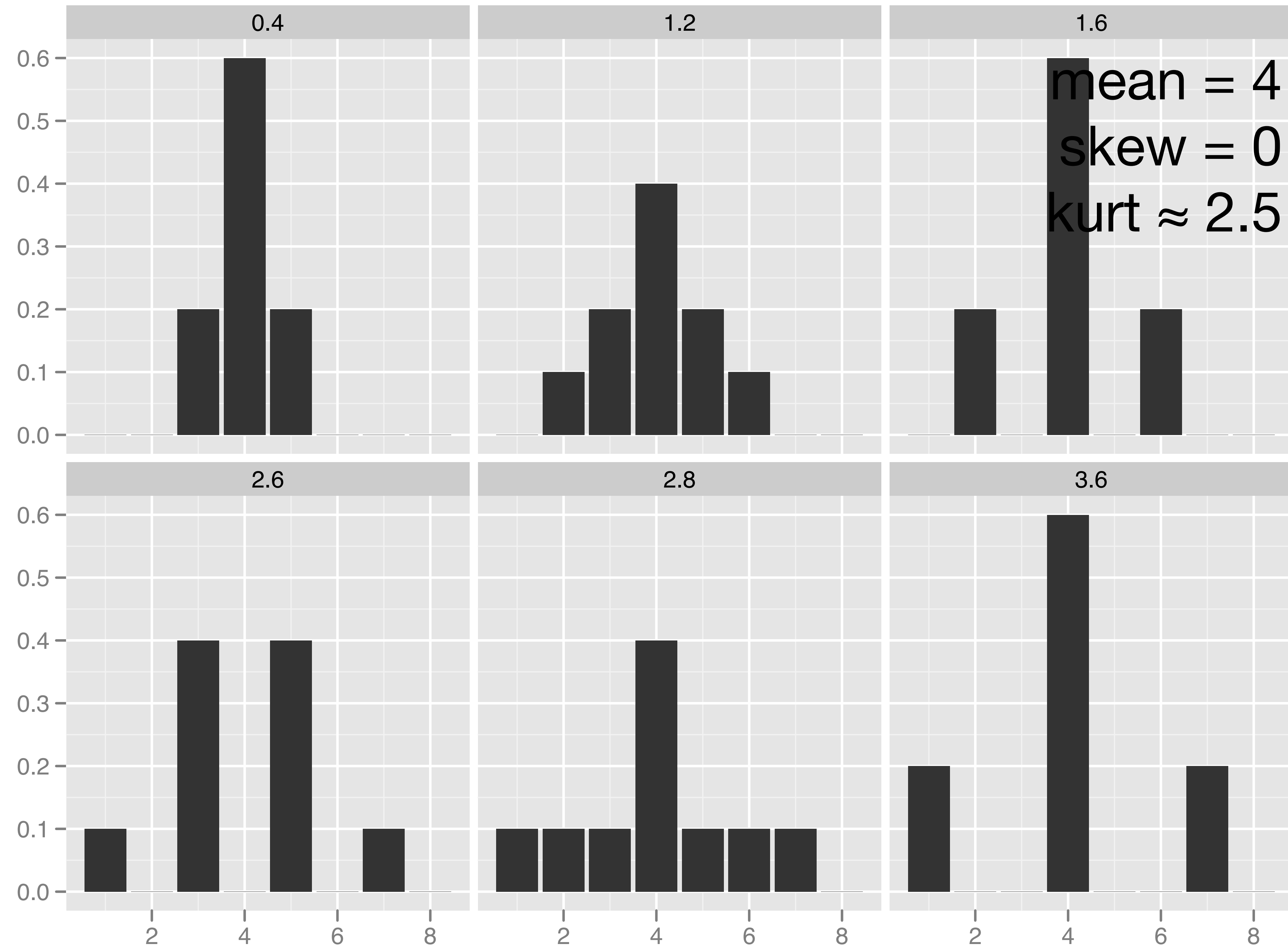
Moments

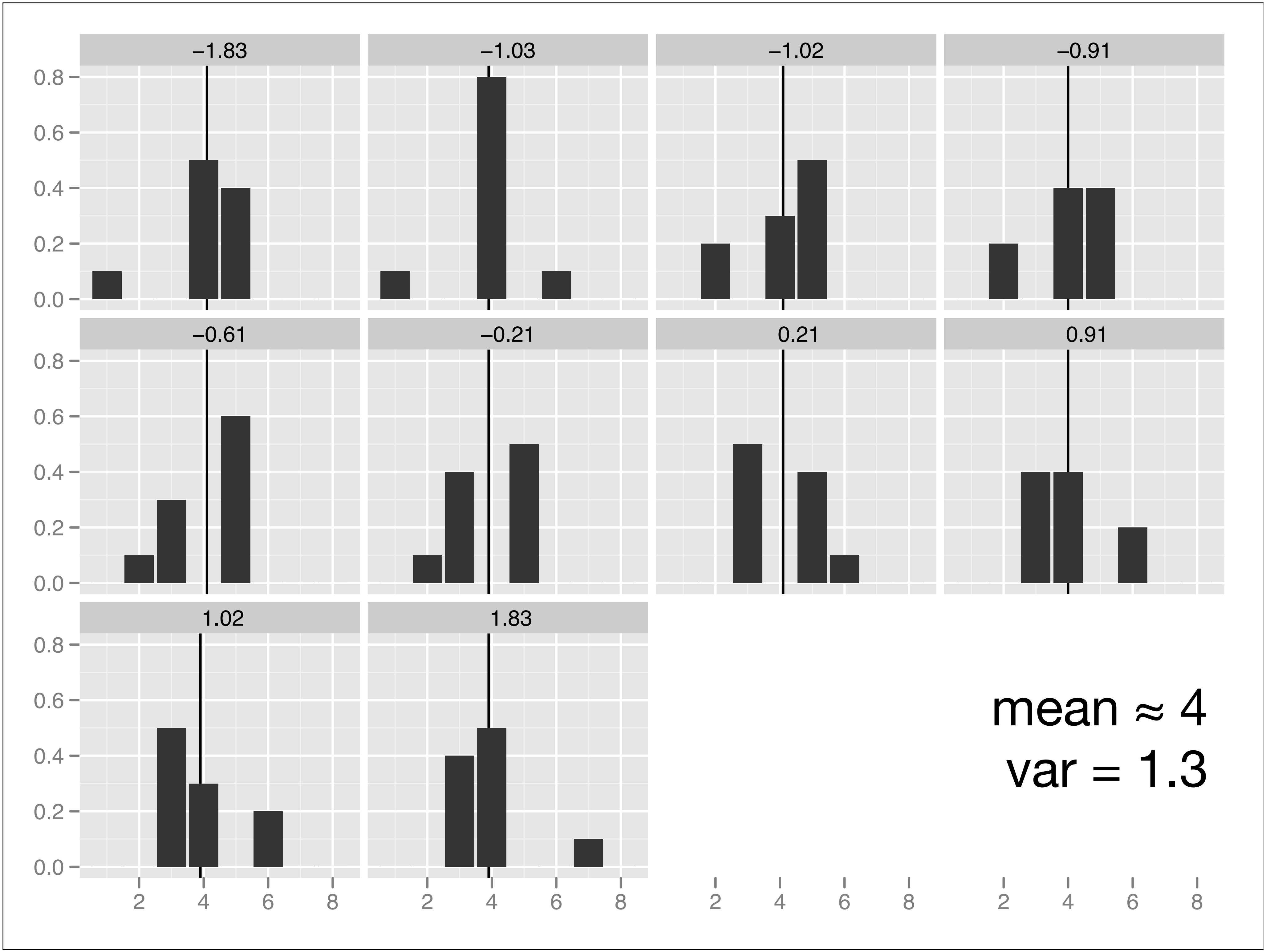
The ith **moment** of a random variable is defined as $E(X^i) = \mu'_i$. The ith **central moment** is defined as $E[(X - E(X))^i] = \mu_i$

The mean is the _________ moment. The variance is the _________ moment.

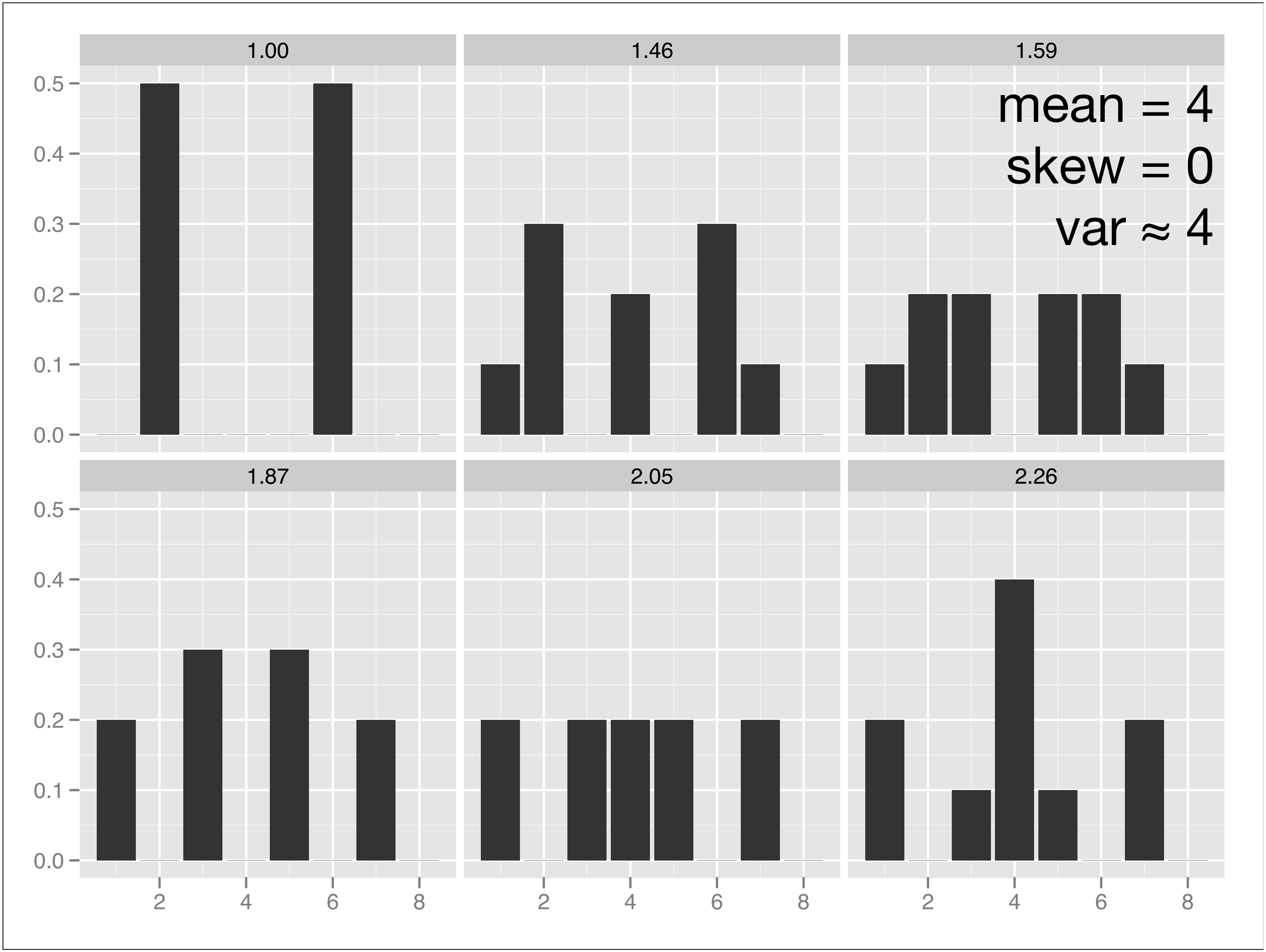
	Name	Symbol	Formula
1	mean	μ	μ'_1
2	variance	σ^2	$\mu_2 = \mu'_2 - \mu^2$
3	skewness	α_3	μ_3 / σ^3
4	kurtosis	α_4	μ_4 / σ^4


3
4
5
6
var =1
skew = 0
kurt = 3.4
0.0
0.1
0.2
0.3
0.4
0.5
0.6
2
4
6
8


0.4
1.2
1.6
2.6
2.8
3.6
mean = 4
skew = 0
kurt ≈ 2.5
0.0
0.1
0.2
0.3
0.4
0.5
0.6
2
4
6
8



−1.83
−1.03
−1.02
−0.91
−0.61
−0.21
0.21
0.91
1.02
1.83
0.0
0.2
0.4
0.6
0.8
2
4
6
8
mean ≈ 4
var = 1.3


1.00
1.46
1.59
1.87
2.05
2.26
mean = 4
skew = 0
var ≈ 4
0.0
0.1
0.2
0.3
0.4
0.5
2
4
6
8

mgf

The **moment generating function** (**mgf**) is $M_X(t) = E(e^{Xt})$
(Provided it is finite in a neighbourhood of 0)

Why is it called the mgf? (What happens if you differentiate it multiple times).

Useful property: If $M_X(t) = M_Y(t)$ then X and Y have the same pmf.

Plus, once we’ve got it, it can make it much easier to find the mean and variance

Expectation of binomial (take 2)

Figure out mgf.
(Random mathematical fact: binomial theorem)

Differentiate & set to zero.

Then work out variance.

Your turn

Compute mean and variance of the binomial. Remember the variance is the 2nd central moment, not the 2nd moment.

Poisson

3.2.2 p. 119

Poisson distribution

X = Number of times some event happens

(1) If number of events occurring in non-overlapping times is **independent**, and

(2) probability of exactly one event occurring in short interval of length h is $\propto \lambda h$, and

(3) probability of two or more events in a sufficiently short internal is basically 0

Poisson

$X \sim \text{Poisson}(\lambda)$

Sample space: positive integers

$\lambda \in [0, \infty)$

Examples

Number of calls to a switchboard

Number of eruptions of a volcano

Number of alpha particles emitted from a radioactive source

Number of defects in a roll of paper

Example

On average, a small amount of radioactive material emits ten alpha particles every ten seconds. If we assume it is a Poisson process, then:

What is the probability that no particles are emitted in 10 seconds?

Make sure to set up mathematically.

mgf, mean & variance

Random mathematical fact.

Compute mgf.

Compute mean & 2^{nd} moment.

Compute variance.

Next week

Repeat for continuous variables.

Make **absolutely sure** you have read 2.5 and 2.6. (hint hint)

Feedback