Stat310

Bayes Rule

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Practice quiz

Don’t worry - it’s just a practice.

You have until 1:10.

Don’t hand it in.

Homework

Due by the end of class!

(Or if you've forgotten it, you have until 9am tomorrow morning to slip it under my office door - DH 2058)

Will probably be an additional help session Tuesday at 4pm. Fill out doodle form if you haven't already.

1. Basic properties of probabilities
2. Working with conditional probabilities
3. Bayes’ rule & natural frequencies
4. Toolbox
5. Intro to random variables
6. For next time

Basics

$P(A') = 1 - P(A)$

If $A \subset B$, then $P(A) \leq P(B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

You should be able to see these from a venn diagram, and if mathematically inclined prove them.

Using $P(A \mid B)$

Assume that A and B are independent.
Show that A and B' are independent.

In homeworks and exams, correctly turning a word problem into a math problem will often get you a few points.

Convert to math

Imagine you select a person at random and give them an HIV test. If the HIV test is positive what's the probability that they have HIV?

If you have HIV, the probability that the test is positive is 99.9%

If you don't have HIV, the probability that the test is negative is 99.99%

In Texas, about 13 people per 100,000 have HIV.

Events

T+ = {test is positive}

T- = {test is negative}

H = {has HIV}

$P(T+ \mid H) = 0.99$

$P(T- \mid H') = 0.9999$

$P(H) = 0.00012$

We want to find $P(H \mid T+)$

Rearranging...

Bayes' rule

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

(Often need to use law of total probability down here)

Alternative approach

Use natural frequencies. Pick a large number of people and break into categories.

Is much more intuitive, and helps you understand why this surprising result is true.

Toolbox

Complements

Convert union to sum

Convert intersection to conditioning

Convert intersection to product
(if independent)

Use law of total probability

Switch conditioning (Bayes' rule)

Random variables

Intro to rv's

Probability is a set function. Kind of tricky to deal with. Easier to deal with functions of numbers.

Want to ignore details of problem (e.g. specific events) and focus on essence.

Real world ➙ mathematical world

Definitions

A **random variable** is a random experiment with a numeric sample space. Usually given a capital letter like X, Y or Z.

(More formally a random variable is a function that converts outcomes from a random experiment into numbers)

The **space** (or **support**) of a random variable is the range of the function (cf. sample space)

Definitions

If the size of the support is **finite** or **countably infinite**, then the random variable is **discrete**.

If the size of the support is **uncountably infinite**, then the random variable is **continuous**.

Your turn

The random experiment is to go to Vegas with $100 and play blackjack until you make $1,000 or lose all your money.

How many different random variables can you generate from this random experiment? Brainstorm in pairs.

Next week

We're going to start working with sums and a little bit of calculus.

http://tutorial.math.lamar.edu/Classes/CalcI/SummationNotation.aspx

Basic calculus: differentiation & integration of (e.g.) polynomials

Read section 2.5